Inflationary Reheating and Fermions

Patrick B. Greene

Department of Physics, University of Toronto,
60 St. George Street, Toronto, Ontario M5S 1A7, Canada

Abstract. Coherent oscillations of the inflaton field at the end of inflation can parametrically excite fermions in much the same way that bosons are created in preheating. Although Pauli-blocking prohibits the occupation number of created fermions from growing exponentially, fermion production occurs in a manner significantly different from the expectations of simple perturbation theory. Here, I discuss the nature of fermion production after inflation and possible applications including the efficient transfer of inflaton energy and the production of super-massive fermions during fermionic preheating.

Consider a simple model of chaotic inflation with the potential $\frac{1}{4}\lambda\phi^4$ for an inflaton field ϕ coupled to a massless spin-$\frac{1}{2}$ field ψ by a Yukawa interaction. At the end of inflation, the inflaton field will oscillate coherently about the minimum of its effective potential with an initial amplitude $\phi_o \sim O(0.1M_p)$. In perturbation theory, one treats the homogeneous and quasi-classical inflaton field as a condensate of scalar inflaton particles, each of which can decay into a pair of ψ-particles. Each fermion then carries away half of the energy of a typical inflaton particle, giving a spectrum narrowly peaked around the comoving momentum $k \approx 0.42\sqrt{\lambda}\phi_o$. The inflaton energy is transfered to fermions after $O(\frac{1}{h^2}) \gg 1$ inflaton oscillations.

To investigate fermion production non-perturbatively, we are interested in the equation of motion for the field operator ψ. Following the usual prescription (see [1] and references therein for details), one seeks eigenfunctions of the Dirac equation in the presence of the classical time-dependent source, $\phi(t)$. As the inflaton field is spatially homogeneous, only the temporal part of the eigenmode obeys a non-trivial equation of motion. These modes, $X_k(\tau)$, obey an oscillator-type equation with a *complex* frequency that varies periodically with time:

$$X_k'' + \left(\kappa^2 + qf^2 - i\sqrt{q}f'\right)X_k = 0 \ . \tag{1}$$

Here, the comoving momentum k enters the equation in the combination $\kappa^2 \equiv \frac{k^2}{\lambda\phi_o^2}$ and the character of the solutions is defined by the parameter $q \equiv \frac{h^2}{\lambda}$ (h is the dimensionless Yukawa coupling). The background oscillations enter in the form

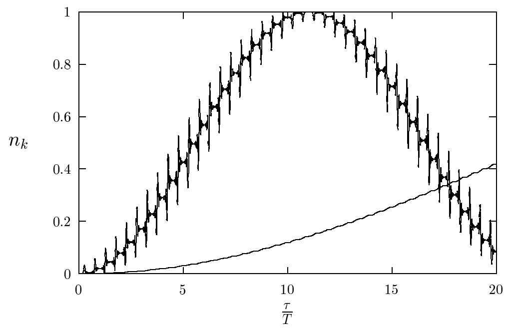

FIGURE 1. *The comoving occupation number n_k of fermions in $\lambda\phi^4$-inflation as a function of time (in units of inflaton oscillations) for $q \equiv \frac{h^2}{\lambda} = 10^{-4}$ and 100 and $\kappa^2 = 0.18$ and 11.9, respectively. The period of the modulation $\frac{\pi}{\nu_k T}$ is about 88 and 22 accordingly.*

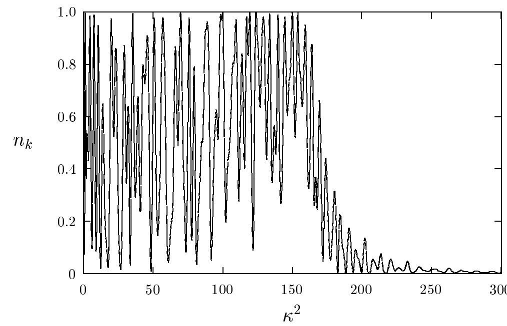

FIGURE 2. *The comoving occupation number of fermions in $m_\phi^2\phi^2$-inflation as a function of (scaled) comoving momentum after 50 inflaton oscillations for initial resonance parameter $q_o = 10^3$. Expansion destroys the details of the resonance band and leads to a fermi-sphere of width $q^{1/4} \sim q_o^{1/4}a^{1/4}$.*

$f(\tau) = cn\left(\tau, \frac{1}{\sqrt{2}}\right)$ having unit amplitude and a period $T = 7.416$. Note that we are working in scaled conformal field and time variables so the effects of expansion do not appear in the equations of motion. We can express the comoving occupation number of particles in a given state through the solutions of eq. (1)

$$n_k(\tau) = \frac{(\Omega_k - \sqrt{q}f)}{2\Omega_k}[\ |X_k'|^2 + \Omega_k^2|X_k|^2 - 2\Omega_k Im(X_k X_k'^*)\ ], \tag{2}$$

where $\Omega_k^2 \equiv \kappa^2 + qf^2$. The energy density of created fermions is $\epsilon_\psi = \frac{1}{2\pi^3}\int d^3k\, \Omega_k\, n_k$.

It turns out that, for all κ^2 and q, the solutions of eq. (1) are periodic in time. This is shown by the numerical solutions for the comoving occupation number in Fig. (1). Furthermore, it is easy to show that the comoving occupation number defined by eq. (2) will obey $n_k \leq 1$ in accordance with the Pauli principle. We see from Fig. (1) that, while the occupation number exhibits some high frequency oscillations (period $< \frac{T}{2}$), the most interesting behavior occurs over longer periods. If we average the occupation number over an inflaton period, $\bar{n}_k(\tau) = \frac{1}{T}\int_\tau^{(\tau+T)} d\tau n_k(\tau)$, the average occupation number is found to obey the simple equation: $\bar{n}_k(\tau) = F_k \sin^2\nu_k\tau$. For a given theory, i.e. for a given value of the resonance parameter q, $F_k \leq 1$ is a momentum dependent amplitude and ν_k is a momentum dependent frequency. In Fig. (3) the amplitude F_k as a function of κ^2 is plotted for several values of q. We see that the perturbative expectation is only met for $q \leq 10^{-4}$.

In fact, for large q, the fermions are excited up to $\kappa^2 \simeq \sqrt{q}$. This is the same result as for the broad bosonic resonance and can be understood in the same manner. The comoving occupation number, eq. (2), is an adiabatic invariant of the mode equation (1). The condition that the modes X_k evolve non-adiabatically is $\dot{\Omega}_k \geq \Omega_k^2$

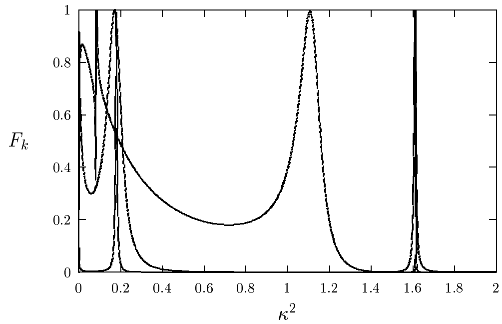

FIGURE 3. *The envelope functions F_k showing the bands of fermion resonance excitation in $\lambda\phi^4$-inflation for $q \equiv \frac{h^2}{\lambda} = 10^{-4}, 10^{-2}$, and* 1.0 *(the narrowest to broadest band, respectively).*

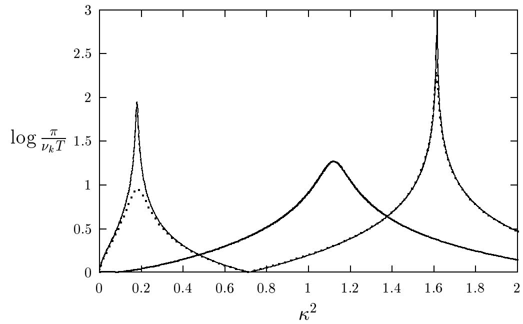

FIGURE 4. *The* log *of the period of modulation $\frac{\pi}{\nu_k T}$ (in units of inflaton oscillations) as a function of κ^2 for $q \equiv \frac{h^2}{\lambda} = 10^{-4}, 10^{-2}$, and* 1.0 *for the light, dotted, and heavy curves respectively.*

which leads to the condition $\kappa^2 \leq \sqrt{q}$ for non-adiabatic evolution, and thus, particle creation. This occurs much more rapidly than one would expect from perturbation theory. In Fig. (4) the period of mode oscillations $\frac{\pi}{\nu_k T}$ is plotted in units of inflaton oscillations. For all q, the bands saturate after only $10 - 100$ inflaton oscillations.

Turning to the energetics for the most interesting case of the broad resonance excitation, $q \gg 1$, we find $\epsilon_\psi \sim 0.1 h^2 q^{1/4} \epsilon_\phi$, where the inflaton energy is $\epsilon_\phi = \frac{1}{4}\lambda\phi_o^4$. In chaotic $\frac{1}{4}\lambda\phi^4$-inflation, $\lambda \simeq 10^{-13}$. If the resonance parameter q is large but the coupling parameter is small, $h \leq 0.1$, only a small fraction of the inflaton's energy will be converted into fermions. Although explosive decay will not occur for this model, more general theories such as hybrid models can have large enough resonance parameters to allow efficient decay to fermions.

If the fermion field has a small bare mass term or if we consider inflation with a $\frac{1}{2}m_\phi^2\phi^2$ potential, the conformal invariance of the theory will be broken. In this case, the occupation number of fermions for $q \gg 1$ no longer evolves periodically but becomes stochastic, rapidly fluctuating between $n_k = 1$ and $n_k = 0$. This destroys the well defined resonance bands depicted in Fig. (3) and can lead to the production of super-massive fermions of mass $m_\psi \leq h\phi_o$. These fermions can come to dominate the energy density of the universe or survive as massive relics.

As a particular example of the changes brought by expansion, consider $m_\phi^2\phi^2$-inflation with a Yukawa coupling to a still massless fermion. In the broad resonance case, one gets a sphere in comoving momentum space with average occupation $\bar{n}_k = \frac{1}{2}$ that *expands* with time. A snapshot of this sphere is shown in Fig. (2).

REFERENCES

1. Greene P. B., and Kofman L., to appear in *Phys. Lett.* **B448**, (1999); Available as eprint hep-ph/9807339.